



Credit Risk : Valuation, Quality Migration, and KMV Approach

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Abstract: In this paper, we show how CreditMetrics model quantifies the risk of credit loss resulting from modification of the creditworthiness of borrowers. Two measures are used by CreditMetrics in the appreciation of credit risk: the standard deviation and the percentile level. CreditMetrics approach estimates credit correlation from bond prices and credit spreads. It estimates the correlation in the movements of the credit spreads. The approach of KMV for credit valuation is based on option pricing model of Merton, and can then be applied to detect potential deterioration of credit using actual market data.

Keywords: Credit Risk Valuation ; Credit Quality Migration ; KMV Approach ; CreditMetrics Approach.

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1. Introduction

Credit risk indicates the risk of economic loss due to the failure of the counterparty to meet its contractual obligations. The credit risk can be estimated using the cost of replacing cash flows if the other party defaults. Credit risk involves the probability of non-payment, either on a future obligation or a current transaction.

Credit risk analysis and credit valuation need some preliminary definitions of the basic concepts. Drivers of credit risk are default, exposure at default and the loss given default. Altman (1987) defines a *default* event with reference to missed interest and principal payments. The *default probability* refers to the likelihood that an obligor will encounter credit distress within a given period. *Exposure at default*, EAD, also known as credit exposure, refers to the amount which is subject to changes in value upon a change in credit quality. EAD can be interpreted as the maximal loss in the event of default. *Loss given default* (LGD) stands for the estimated loss under the event of default. Credit risk analysis and credit valuation requires, in addition, the construction of the distribution of probabilities of loss given default in a portfolio context. The expected loss corresponds to the average shortfall given that a loss occurs or exceeds a given level. The *counterparty* is the partner in a transaction in which each

side takes broadly comparable credit risk to the other. The *current exposure* corresponds to the amount it would cost to replace a transaction now if the counterparty defaults. *Credit quality migration* refers to the possibility that an obligor with a certain credit rating migrates to any other credit rating by the risk horizon. The organisation of the paper is as follows. The next section presents the credit risk measures, the CreditMetrics computation of the credit risk both for a single exposure and for a portfolio of exposures. The third section shows how the CreditMetrics approach estimates the credit quality migration. The fourth section examines the credit valuation in the KMV approach.

2. Credit risk and credit valuation

CreditMetrics model quantifies the risk of credit loss resulting from modification of the creditworthiness of borrowers. Initiated by the JP Morgan bank on 1997, this approach has been used by many banks and financial institutions. The model is marked-to-market since the estimated chances of default are obtained from the rating changes. The volatility of the value of a bond or a portfolio is explained by the chances of default, downgrading, and upgrading.

2.1 Credit risk measures

Two measures are used by CreditMetrics in the appreciation of credit risk: the standard deviation and the percentile level. These measures reflect potential losses from the same portfolio distribution. The first measure (determination of the standard deviation) needs the computation of the mean value for the portfolio by multiplying the values with the corresponding probabilities and then adding the resulting values. This allows the computation of the standard deviation. The second measure is a specified percentile level which indicates the lowest value that the portfolio will achieve 1 % of the time (the 1st percentile). The likelihood that the actual portfolio value is less than this number is only 1 %. These measures apply to exposures like loans, letters of credit, forward contracts and swaps.

2.2 Stand alone or single exposure risk estimation

Credit risk can be defined as the volatility of its value. CreditMetrics computes the credit risk for a single or stand-alone exposure on three steps.

The first step: Credit rating migration

This step estimates changes in value due to up (down) grades and default. The likelihood of credit rating migration is conditioned on the senior unsecured credit rating of the obligor. A transition matrix is conceived using public rating migration data. This allows the determination of the likelihoods of migration to any credit quality state in a given period.

The second step: Valuation

The values at the risk horizon are determined for the credit quality states. When the credit quality migration corresponds to a default case, the likely residual value is a function on the seniority class of the debt. When the credit quality migration is in another category, the forward zero curves for each rating category are used to revalue the bond's remaining cash flows.

The third step: Credit risk estimation

Using the likelihoods and values, it is possible to calculate the risk estimate: the standard deviation or the percentile level.

2.3 The portfolio context of credit

Portfolio diversification provides a mean to reduce the likelihood of losses in credits. A quantitative portfolio approach to credit risk management allows the study of concentration risk. This risk refers to additional risk resulting from higher exposure to one or several correlated obligors. A portfolio credit

risk methodology, as the one in CreditMetrics, allows to capture simultaneously the benefits of diversification and concentration risks and provides an efficient risk-based capital allocation process. If we consider a bond rated BBB which matures in n periods, then at the end of the year, the bond stays at BBB, the issuer defaults or it migrates up or down to one of the other categories. Hence, the probabilities that this bond will end up in one of the other categories in a period allow the computation of the bond price under each of the possible rating scenarios. The new present value of the bond can then be calculated from the remaining cash flows under its new ratings. The discount rate is obtained from the forward zero curve which is different for each rating category. The knowledge of the probabilities or likelihoods for the bond to be in a given rating category and the values of the bond in these categories allows the determination of the distribution of value of the bond in one period.

2.4 Portfolio risk estimation in CreditMetrics

The procedure for a stand-alone exposure can be used for a portfolio of two exposures. In the presence of two obligors, all possible combinations of states can contribute to the joint risk. When the two obligor's credit rating changes are independent, the joint likelihoods are the product of the individual likelihoods. The introduction of correlations needs a model that links firm asset value to firm credit rating. The Merton's (1974) model or the option theoretic valuation of debt is useful in this context. When the assets value falls in a way such that the value is less than the amount of liabilities outstanding (the default threshold), then the firm will default. Merton's (1974) model can include rating changes as well as default threshold. The value of the firm's assets with respect to these thresholds determines the future rating. The extension of this model allows a link between the firm's value and the firm's credit rating. It allows to build the joint probabilities for different obligors.

2.4.1 Portfolio credit risk

The *volatility* of value due to credit quality changes can be calculated from the joint states between obligors, i.e, the joint likelihoods and revaluation estimates. The value of the portfolio is simply the sum of the individual values. The standard deviation and the percentile level are calculated as in the stand-alone exposure case.

The *marginal risk* indicates where the risks are concentrated in the portfolio. The marginal risk shows the marginal increase to the portfolio risk as a consequence of adding a new bond to the portfolio. The marginal risk can refer also to the marginal impact on portfolio risk of increasing the exposure by a small amount.

2.4.2 Differing exposure type

CreditMetrics determines the credit risk for market-driven instruments like swaps and forward contracts. The value of a swap is given by the difference between two components. The first corresponds to the forward risk-free value of the swap cash flows. This component is the same for all forward credit rating states. The second component corresponds to the loss expected on a swap, resulting from a default net of recoveries by the counterparty on all the cash flows after the risk horizon. The difference between this second component and the first one allows the revaluation of the swap. The revaluation of the swap is based on the following formula:

$$\begin{array}{l} \text{Value of the swap} \\ \text{in a period (with rating } R) \end{array} = \begin{array}{l} \text{Risk free value in one period} \\ \text{through maturity (with rating } R) \end{array} - \begin{array}{l} \text{Expected loss in period 1} \\ \text{through maturity (with rating } R) \end{array}$$

where R refers to any credit rating category. The expected loss for each forward non-default credit rating is given by:

Expected loss (with rating R) = Average exposure (from period 1 to maturity)
 × Probability of default in period 1 through maturity (with rating R)
 × (1 – Recovery fraction)

The probability of default for each rating category between year 1 and the maturity permits to calculate the expected loss. It is calculated using a transition matrix. This method allows the computation of the swap value in each of the non-default credit rating categories. In the case of default during the risk interval, the expected loss in the defaulted state is given by:

Expected Loss (case of default) = Expected exposure (in the first period) × (1 – Recovery fraction)

This expression assumes that the risk interval is very short (one year for example).

3. Default and credit quality migration in the CreditMetrics approach

3.1 Estimating credit quality correlations through bond spreads

It is possible to study price histories of corporate bonds in order to estimate credit correlations from historical data. The correlations in the bond price dynamics allow for estimations of correlations of credit quality moves. This approach estimates credit correlation from bond prices and credit spreads. It estimates the correlation in the movements of the credit spreads. Then, it uses a model to link spread movements to credit events. A pricing model as the one proposed by Duffee (1995) allows to infer the probability of the issuer defaulting from the observed bond spread. This procedure is similar to that of estimating the implied volatility from option prices.

3.2 Asset value model

CreditMetrics specifies a process which drives credit rating changes and the estimation of its parameters. Their approach is based on Merton's (1974) model in which the firm's asset value drives the credit rating changes and defaults. The model assumes that there is a certain level such that if the firm's assets fall below it in the next period, the firm will be unable to satisfy its obligations and will default. The method relates the asset value in one period to the credit rating or default of the firm in one period. The asset values corresponding to changes in rating are referred to as asset value thresholds. The CreditMetrics model assumes that the asset has a mean μ and a volatility σ . This parameterization of the asset value process allows a connection between the asset thresholds and the transition probabilities for a given firm. It also allows the computation of the probability that each event occurs. Hence, for example:

$$\Pr(\text{default}) = \Pr(R < Z_{\text{def}}) = \Phi(Z_{\text{def}}/\sigma)$$

where Φ is the cumulative distribution for the standard normal distribution. Hence, there exists asset returns thresholds Z_{def} such that if R is less than Z_{def} , the obligor will default. If we know $\Phi(Z_{\text{def}}/\sigma)$, it is possible to obtain Z_{def} using $\Phi^{-1}(p)$ which gives the level below which a standard normal variable falls with probability p . This procedure is used to determine threshold values for asset return as well as the corresponding transition probabilities for each obligor according to its asset value processes.

3.3 Application of model outputs

The literature on credit pricing includes Das and Tufano (1996), Jarrow and Turnbull (1995), Merton (1974), etc. The measures of credit risk can have several applications.

Prioritizing risk reduction actions:

When a manager sets priorities for actions, the use of risk statistics allows the reduction of portfolio risk.

Credit risk limits:

The risk statistics can be used for limit setting based on percentage risk. This needs the choice of a specific risk measure to be used. The analysis is concerned with individual exposures and relative measures. The standard deviation is an adequate statistic since it could capture the relative risks of different instruments.

Economic capital assessment:

Credit risk measures can be used to assess the capital which a firm puts at risk by holding a credit portfolio. The analysis is concerned with a portfolio measure. A percentile level can be an appropriate indicator of economic capital. For example, using the 1st percentile level, the economic capital can be regarded as the level of losses on a portfolio for which we are 99 % certain. If the investor is 99 % certain of meeting the obligations in the next period, then the 1st percentile level can be considered as the economic capital allocated to an asset portfolio.

4. Credit Valuation in the KMV approach

Credit valuation provides a mean to reduce the likelihood of losses through portfolio diversification. The credit risk is in general measured in terms of probabilities using a credit valuation model. The model is based on a theory that describes the link between the attributes of the borrowing institution and its possible bankruptcy. The approach of KMV for credit valuation is based on option pricing theory. If the model captures the relationship between the state of the institution and the probability of default, it will account for the borrower's credit standing through time. The model can then be applied to detect potential deterioration of credit using actual market data. Since the firm's ability to pay its debt depends upon its future market value, the firm can easily raise cash by selling off a fraction of its assets or by issuing debt. The value of the firm depends on the present assessment of its future returns from its business. This value can be obtained from the value of equity and that of bond issues. The value of equity corresponds to the product of share prices by the number of shares outstanding. The value of bond issues is given by the current price (per unit of face value) times the face amount of the issue. Current liabilities are in general valued at their nominal values.

5. The credit risk literature

5.1 The statistical prediction of the likelihood of default

Accounting analytic methods estimate firm specific credit quality with respect to financial ratios. Moody's, S&P, and Fitch Ratings publish historical default likelihoods for their letter rating categories. Several models try to build credit quality estimation with reference to statistical techniques. The main approaches used within this context are qualitative dependent variable models, discriminant analysis and neural networks. The linear discriminant analysis tries to categorize between the firms which have defaulted and the others. A classification is done with respect to a statistical estimation approach. This category includes for example the Z-Score models a la Altman. Several other methods can be used. Examples include logistic regressions, probit/logit analysis, etc. Neural network techniques have also been applied to credit scoring.

5.1 The option pricing approach

This approach is initiated mainly by Merton (1974). The firm's value evolves randomly and default occurs when the value of the firm's assets becomes less than its obligations. This method is applied by KMV. The model gives a continuous numeric value which may be mapped to default likelihoods. The reader can refer to Das (1995) and Hughston (1996). Jarrow, Lando and Turnbull (1994) assume that credit ratings follow a Markov chain and use a matrix of ratings transitions to model default. This model uses a fixed recovery rate. Das, Sanjiv and Tufano (1996) extend the analysis to a context where the recovery rate is a random variable.

6. Conclusion

A default event implies missed interest and principal payments. Credit quality migration measures the possibility that an obligor with a certain credit rating migrates to any other credit rating by the risk horizon. The standard deviation and the percentile level apply to exposures like loans, letters of credit, forward contracts and swaps. Portfolio diversification provides a mean to reduce the likelihood of losses in credits. The expected loss corresponds to the average shortfall given that a loss occurs or exceeds a given level. CreditMetrics approach has been used by many banks and financial institutions. The approach of KMV for credit valuation uses the option pricing model of Merton. Moody's, S&P, and Fitch Ratings publish historical default likelihoods for all rating categories.

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